Econometrics Models for Copper Recovery: A Case Study of North Waziristan-Copper Deposits

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(Received on 30th December 2009, accepted in revised form 17th April 2010)

Summary: Fourteen econometrics models have been developed to evaluate the effects of various flotation process variables like, Propylxanthate (X_1 g/tonne), pH (X_2,) Sodium Cyanide (X_3 g/tonne), Sodium sulphide (X_4 g/tonne), Frother (X_5 g/tonne), Pulp density (X_6 w/vol), and Conditioning time (X_7 minute) on the copper recovery Y_R North Waziristan-NWFP Pakistan. Ordinary Least Square OLS method has been applied as an analytical technique for regression analysis. It has been concluded in this study that model given in equation 7 is best model among all. This equation shows that with the increase of one unit of X_1, Y_R will increase 0.05 units keeping all other variables constant.

Introduction

Indeed, copper is considered very indispensable minerals for boosting industries in a country. A number of chemical industries use copper metal the important commercial use of copper is in the manufacturing of electrical instruments/ apparatus and wire. Besides copper used extensively as roofing, in making copper utensils, and for coins *etc.* Copper tubing is used in plumbing, because of its high heat conductivity, in heat-exchanging devices such as refrigerator, and air-conditioner coils. Similarly copper in powder form sometimes used as a pigment in paints. Copper is needed by a variety of key systems in the body. Numerous enzymes necessary for reproduction, immunity, and growth need copper. In addition, copper is necessary for proper metabolism of iron, maintenance of connective tissue, pigmentation of skin and hair, maturation of hoof tissue, and many other functions.

Federally Administrated Tribal Areas (FATA) Development Corporation Pakistan carried out exploration work in Shinkai and Degan area of North Waziristan-NWFP Pakistan. According to the study in the mentioned area almost 1.5 million tonnes of estimated reserve of copper ore were found [1-3]. It is envisaged that statistical approach was reported earlier to predict the most optimal conditions for flotation of North Waziristan copper deposits.

A classical first-order kinetic model has developed, combined with a properly built statistical model. Statistical approach has been reported to evaluate the lead mineral selectivity by various collectors' types in floatation [4-5]. In copper ore concentration process, an original stochastic mathematical model (steam model) is proposed to minimise copper losses in tailings [6]. Mathematical models were developed for copper powder metallurgy (P/W) working process, using regression analysis and analysis of variance (ANOVA) in order to study the main and interaction effects of process parameter [7]. Recently a mathematical model has been developed for the particle–bubble flotation complex motion in vibration flotation [8].

Results and Discussion

Figs. 1 and 2 shows that empirical results of the regression equations with single predictors for recovery of copper obtained from least square analysis are as follows:

$$Y_R = 41.4 + 0.7\,X_3 \quad (1)$$

$$Y_R = 61.07 + 0.089\,X_4 \quad (2)$$

The R^2 show that the first equation explained 85.2% of the variation and the second equation explained 67.3% variation in the recovery

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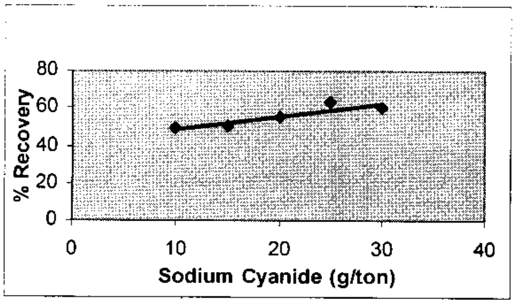

Fig. 1: Effect of Sodium Cyanide (X_3) on the Recovery of Copper.

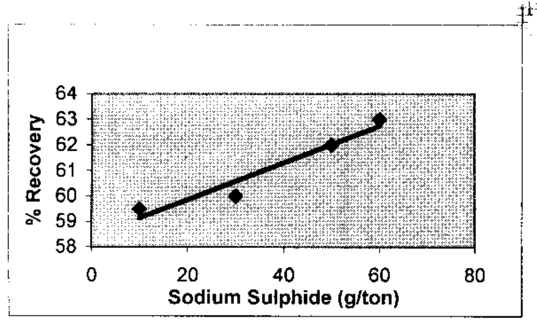

Fig. 2: Effect of Sodium Sulphide (X_4) on the Recovery of Copper.

of copper using flotation process. Equation (1) is based on combined data from seven experiments.

In a single predictor variable models involving X_3 and X_4 were found as better model as the first model show that 30 g/tonne of depressant gave maximum recovery 71.3% and Recovery increased at the rate 0.7% per one-gram increase in depressant. The second model show that recovery increased at the rate of 0.089% per one gram increase in sulfidizer.

Among the twenty-one models in the subset with two predictor variables, the two best regression equation involving two predictor variables are:

$$Y_R = 38.5 + 0.837\ X_3 + 0.168\ X_4 \quad (3)$$

$$Y_R = 50.7 + 0.991\ X_3 - 0.164\ X_5 \quad (4)$$

The equation (3) involving X_3 and X_4 explained 89.2% and the equation (4) involving X_3 and X_5 explained 87.5% of the variation in the recovery of copper.

Response surfaces were developed for the variables involved in the above two equations.

Fig. 3 depicts the combine response surface for sodium cyanide (X_3 g/tonne) and depressant sodium sulphide (X_4 g/tonne) on the recovery of copper reveals that the maximum peak of surface shows the estimated maximum recovery of 73.69 % with 25 gram per tonne of sodium cyanide and 50 gram per tonne of sodium sulphide.

Fig. 4 shows the combine response surface for sodium cyanide (X_3 g/tonne) and frother dosage (X_5 g/tonne) on the estimated recovery of copper given in the Fig. 4. The maximum peak of surface shows the maximum recovery of 76.33% with 30 gram per tonne of sodium sulphide and 60 grams per tonne of frother.

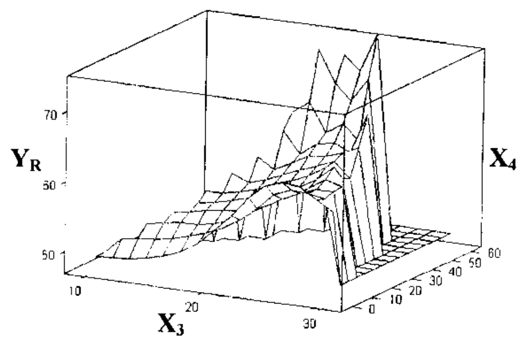

Fig. 3: Copper Recovery (Y_R) Response Surface for Sodium Cyanide (X_3), and Sodium Sulphide Dosage (X_4).

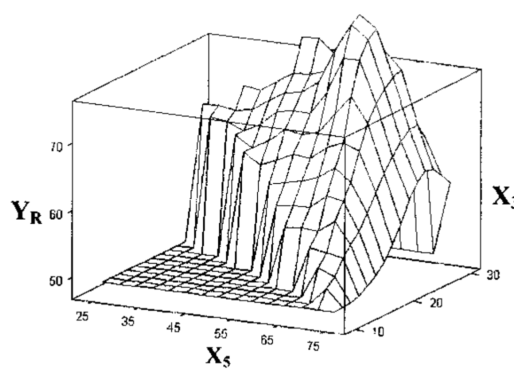

Fig. 4: Copper Recovery (Y_R) Response Surface for Sodium Cyanide (X_3), and Frother Dosage (X_5).

The best subset program picked the following two best regression equations involving three predictor variables among the fifty-five, 3-variables models in the subset with 3-predictors:

$$Y_R = 29.0 + 0.053\ X_1 + 0.782\ X_3 + 0.171\ X_4 \quad (5)$$
$$Y_R = 15.6 + 0.835\ X_3 + 0.185\ X_4 + 0.762\ X_6 \quad (6)$$

The two equations (5) and (6) explained 91.7% and 90.7% of the total variation in recovery of copper.

Among the next subset with 4 predictors, the following two best regression equations involving four predictor variables were selected by the program.

$$Y_R=6.13 + 0.05\ X_1 + 0.78\ X_3 + 0.19\ X_4 + 0.75\ X_6 \quad (7)$$
$$Y_R = - 26.6 + 3.83\ X_2 + 0.741\ X_3 + 0.189\ X_4 + 0.762\ X_6 \quad (8)$$

Equation (7) explained 93.2% of the total variation in recovery; this model all variables are collectively important except intercept. Equation (8) explained 92.4% of the variation in the data for recovery of copper, but the intercept and X_2 are not statistically significant so we drop this model.

The following two best regression equations involving five predictor variables were selected by the program:

$$Y_R = - 35.0 + 0.0534\ X_1 + 3.74\ X_2 + 0.688\ X_3 + 0.191\ X_4 + 0.762\ X_6 \quad (9)$$
$$Y_R = 8.56 + 0.054\ X_1 + 0.778\ X_3 + 0.196\ X_4 + 0.782\ X_6 - 0.305\ X_7 \quad (10)$$

In both equations (9) and (11), the inclusion X_2 and X_7 did not improve the fit significantly.

The improvement in R^2 from equations with five predictor variables (equations 9 and 10) over equations with four predictors (equations 7,8) are very small and not significant, so the models with four predictors sufficiently explained the variation in copper recovery.

The two best regression equations involving six predictor variables are given below:

$$Y_R = -32.6 + 0.053\ X_1 + 3.75\ X_2 + 0.687\ X_3 + 0.200\ X_4 + 0.783\ X_6 -0.306\ X_7 \quad (11)$$

$$Y_R = -29.9 + 0.053X_1 + 3.73X_2 + 0.694X_3 + 0.155X_4 - 0.079X_5 + 0.795\ X_6 \quad (12)$$

There improvements with equations (11) and (12) as compared to equations (9) and (10) are not statistically significant. The full model is given below, the improvement in R^2 is very small over model (11) and (12).

$$Y_R = -25.2 + 0.053X_1 + 3.74\ X_2 + 0.693X_3 + 0.158X_4-0.0982X_5 + 0.833X_6-0.450X_7 \quad (13)$$

From the information provided by the best subset procedure, it is concluded that equations with four predictor variables explains sufficient variation in the recovery of copper ore data from all the experiments. The best equation involves variables sodium propylxanthate, sodium sulphide, sodium cyanide and pulp density.

Experimental

The experiments were carried out in the Department of Mining Engineering, N-W.F.P University of Engineering & Technology, Peshawar to observe the combine effect of X_1 = propylxanthate (g/tonne), X_2 = pH, X_3 = Sodium, Cyanide (g/tonne), X_4 = Sodium sulphide (g/tonne), X_5 = Frother (g/tonne), X_6 = Pulp density (w/vol) and X_7 = Conditioning time (minutes).

Methodology

Primary data used in the present study received from the above experimental work. Fourteen econometric models have been used and such as for regression analysis the method of least OLS has been applied.

Econometrics Models

The total all possible regression models involving seven variables are 128, it is very difficult to check all these models so the best subset procedure was used to select two best models involving one, two, three, four, five and six, variables. Using best subset method by Minitab software, we got thirteen models, two in each subset and the full model for recovery of copper. The summaries of best subset models are given below:

Models with one variable are:

M_1: $Y_R = \beta_0 + \beta_1 X_3$

M_2: $Y_R = \beta_0 + \beta_1 X_4$

Models with two variables are:
M_3: $Y_R = \beta_0 + \beta_1 X_3 + \beta_2 X_4$
M_4: $Y_R = \beta_0 + \beta_1 X_3 + \beta_2 X_5$

Models with three variables are:
M_5: $Y_R = \beta_0 + \beta_1 X_1 + \beta_2 X_3 + \beta_3 X_4$
M_6: $Y_R = \beta_0 + \beta_1 X_3 + \beta_2 X_4 + \beta_3 X_6$

Models with four variables are:
M_7: $Y_R = \beta_0 + \beta_1 X_1 + \beta_2 X_3 + \beta_3 X_4 + \beta_4 X_6$
M_8: $Y_R = \beta_0 + \beta_1 X_2 + \beta_2 X_3 + \beta_3 X_4 + \beta_4 X_6$
M_9: $Y_R = \beta_1 X_2 + \beta_2 X_3 + \beta_3 X_4 + \beta_4 X_6$

Models with five variables are:
M_{10}: $Y_R = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \beta_5 X_6$
M_{11}: $Y_R = \beta_0 + \beta_1 X_1 + \beta_2 X_3 + \beta_3 X_4 + \beta_4 X_6 + \beta_5 X_7$

Models with six variables are:
M_{12}: $Y_R = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \beta_5 X_6 + \beta_6 X_7$
M_{13}: $Y_R = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \beta_5 X_5 + \beta_6 X_6$

Model with seven variables are:
M_{14}: $Y_R = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \beta_5 X_5 + \beta_6 X_6 + \beta_7 X_7$
Where,
M_1, M_2,. M_{14} represents econometrics models; Y_R denotes recovery of copper and $\beta_0, \beta_1, \beta_2 .. \beta_7$ are estimated parameters.

Conclusion

The present work was conducted with the broad aim to evaluate the effects of various floatation factors on copper recovery in North Waziristan-NWFP Pakistan. Results of this present quantitative research are meaningful. It has been concluded that in all fourteen econometrics models, the best-fitted model comprises of the four variables, that is X_1, X_3, X_4, and X_6 and is given as under;

$$Y_R = 6.13 + 0.05\ X_1 + 0.78\ X_3 + 0.19\ X_4 + 0.75\ X_6 \quad (7)$$

The result shows that if we increase one unit of X_1, Y_R will increase 0.05 units where keeping all other variables constant. We can define the other entire coefficient in the same fashion. These coefficients (slopes) give partial value.

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