Optimization of Operating Process Parameters of Copper Flotation by Using Statistical Techniques

[1]SARDAR ALI*, [2]MUHAMMAD MANSOOR KHAN,
[3]MUHAMMAD TUFAIL AND [4]KHWAJA MUHAMMAD YAHYA
[1]Division of Science and Technology, University of Education Lahore, Pakistan.
[2]Department of Mining Engineering, NWFP University of Engineering and Technology, Peshawar, Pakistan.
[3]Gandhara Institute of Science & Technology, PGS Engineering College, Peshawar, Pakistan.
[4]Department of Computer System Engineering, NWFP University of Engineering and Technology, Peshawar, Pakistan.

(Received 26th June 2006, accepted in revised form 23rd May 2007)

Summary: Flotation process parameters were studied to concentrate major chalcopyrite ($CuFeS_2$) sulphide copper mineral of North Waziristan. The important flotation reagents *i.e.* collectors, depressant, dispersant, frother and conditioning time were examined. During stepwise of flotation parameters copper was enriched from 0.9 to 20 % in a single stage cleaning with recoveries over 83 %. In order to have an insight of combined effect of all the parameters on the recovery and grade of the final product, a numbers of experiments are required to be conducted. This approach requires time and resources and still the findings are not reliable. This is due to analysis of experimental data which is based on empirical techniques. Therefore it was suggested that a statistical techniques will be used to analyze the data to project the combined effect of flotation parameters. Hence with the available data of seven parameters Mathematical Models were developed by applying ordinary least square, OLS method for regression analysis and adopted general to simple modeling procedure. Resulting Equation (3) is statistically significant which includes the process parameters type and dosage of xanthate (Propyl Xanthate), depressant (Sodium Cyanide), Sulphidizer, (Sodium Sulphide) and pulp density. This paper comprises of five parts Introduction, discussion, methodology used for modeling and testing of the model, conceptual model, results and discussions and at end conclusions were drawn.

Introduction

Federally Administrated Tribal Areas (FATA) Development Corporation (Pakistan) carried out exploration work for more than 10 years to ascertain 1.5 million confirmed reserves and 122 million tones of estimated reserves [1-2] of copper ore in Shinkai and Degan area of North Waziristan bordering Afghanistan. Experimental work on laboratory and pilot scale was carried out by the Department of Mining Engineering, NWFP University of Engineering and Technology Peshawar [3-5]. It is envisaged that statistical approach will be used, based on the earlier work reported to predict the most optimal conditions for flotation of North Waziristan Copper deposit. This will facilitate in the design of flotation circuits to be used on pilot plant with already determined parameters to save time and expenditure. The simulation of the mineral processing system design optimization of flotation parameters and control is used for the last 32 years [6-10].

Recently a classical first–order kinetic model combined with a properly built statistical model was developed [11] based on a factorial experimental design, in order to predict the rougher flotation efficiency for various flotation conditions. A statistical approach has been reported to evaluate the lead zinc selectivity by various collectors types in flotation. For selection of most suitable collectors of flotation, six replicated tests were used to estimate the statistical population characterized by an average and variance method. Numerous other empirical and phenomenological mathematical models based on various assumptions for flotation were proposed [12-17]. However, no statistical approach has been reported on the North Wazristan Copper ore.

The main focus in this research work is to investigate and analyze the recovery of copper of the North Waziristan (Pakistan) through regression analysis. This study is based on primary data given in Table-1. The representative samples were collected by the Department of Mining Engineering, NWFP University of Engineering and Technology, Peshawar, with assistance of the political authorities of North Waziristan Agency and Federally

*To whom all correspondence should be addressed.

Administered Tribal Area Development Corporation. Mathematical models were developed by applying Ordinary Least Squares (OLS) method for regression analysis and using Microsoft excel and Minitab statistical software for computations in the study.

Strategy Modeling

Using general which simple strategy, Equation 1 is the linear model consists of seven process parameters.

$$Y_R=\beta_0+\beta_1X_1+\beta_2X_2+\beta_3X_3+\beta_4X_4+\beta_5X_5+\beta_6X_6+\beta_7X_7+\varepsilon \quad .....(1)$$

where:

Y_R = Recovery of Copper

X_1 = Collector type and Dosage (g/ton) (Propyl Xanthate)

X_2 = Variation in pH (g/ton)

X_3 = Depressant Sodium cyanide (g/ton) NaCN

X_4 = Sulfidizer (g/ton) Na_2S

X_5 = Frother Dosage(g/ton) (Pine oil)

X_6 = Pulp Density (weight/volume)

X_7 = Conditioning Time (minute)

ε = error term

β_i = i-th parameter where i = 1,2,3,.........,7

Results and Discussion

The equation (1) is the first model, which has been estimated, as given in Table-1, was;

$$t - statistics = \frac{observed\ value - expected\ value}{S.E.}$$

The residuals are independent identically normally distributed. From Table-2 column 4 and 5 p-value and t-statistic of intercept, X_2, X_5 and X_7 suggest that these variables are not statistically significant, therefore, we dropped these insignificant variables.

Figs. 1, 2 and 3 are the conceptual models showing the interdependency of variables for flotation process, while Fig. 3 shows the conceptual model of the significant interdependency variables for flotation process for recovery of copper.

The Fig. 4 shows visual test for standard residuals of seven variables and it has little deviation from 45-degree line yet it does not give vital evidence against the normality, as shown in Table-3.

It is obvious from the histogram Fig. 5 that the distribution of the error terms is symmetric but not normal. Table-4 shows that the co-efficient of skewness for standard residual is –0.47, which is inside the 95 % confidence interval. Thus the data is not skewed and therefore satisfies one of the normality conditions. Also E. Kurtosis is 1.107, which is inside the 95 % confidence interval and hence satisfies normality condition.

Table-1: Experimental Data for Seven Variables for Enrichment of Copper Ore of North Waziristan.

X_1	X_2	X_3	X_4	X_5	X_6	X_7	Y_R
50	11	0	0	75	30	10	32
100	11	0	0	75	30	10	32.7
150	11	0	0	75	30	10	38
200	11	0	0	75	30	10	41.5
250	11	0	0	75	30	10	41
200	10	0	0	75	30	10	35
200	10.3	0	0	75	30	10	36
200	11	0	0	75	30	10	42
200	11.58	0	0	75	30	10	45.2
200	12	0	0	75	30	10	40
200	11.58	10	0	75	30	10	49
200	11.58	15	0	75	30	10	50
200	11.58	20	0	75	30	10	55
200	11.58	25	0	75	30	10	63
200	11.58	30	0	75	30	10	60
200	11.58	25	10	75	30	10	60
200	11.58	25	30	75	30	10	63
200	11.58	25	40	75	30	10	67
200	11.58	25	50	75	30	10	73
200	11.58	25	60	75	30	10	59.4
200	11.58	25	50	25	30	10	70
200	11.58	25	50	46	30	10	74
200	11.58	25	50	70	30	10	71.56
200	11.58	25	50	46	15	10	56
200	11.58	25	50	46	25	10	64
200	11.58	25	50	46	30	10	75
200	11.58	25	50	46	35	10	68
200	11.58	25	50	46	30	10	69.77
200	11.58	25	50	46	30	13	73.3
200	11.58	25	50	46	30	16	68
200	11.58	25	50	46	30	18	64

Table-2: Ordinary Least Squar (OLS) Estimates of Seven Process Parameters

Predictor	Coef	SE Coef	T	P
Intercept	-28.37	25.43	-1.11	0.27
X_1	0.05	0.02	2.57	0.01
X_2	3.74	2.11	1.76	0.09
X_3	0.68	0.10	6.77	0.00
X_4	0.15	0.04	3.25	0.003
X_5	-0.05	0.07	-0.77	0.44
X_6	0.84	0.23	3.62	0.001
X_7	-0.47	0.40	-1.17	0.25
R-square	0.946	F-statistics	59.6	
Adjusted R	0.93	Standard Error	3.678	
Observations	31			

CONCEPTUAL MODELS

Model No. 1

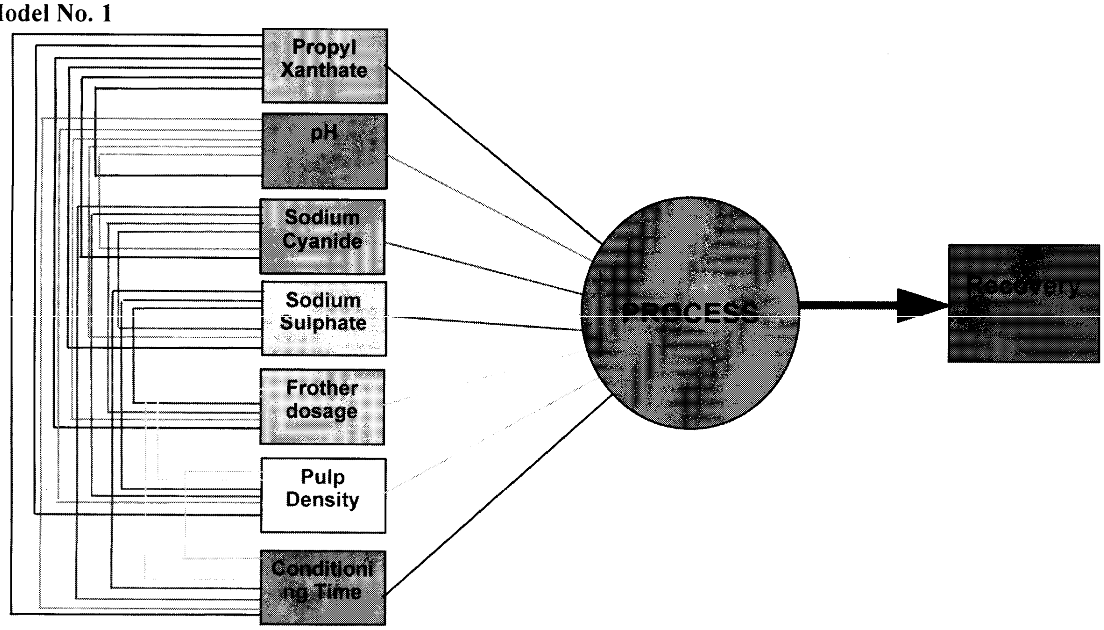

Fig. 1: Recovery model one for enrichment of copper ore of North Wazirstan.

Model No. 2

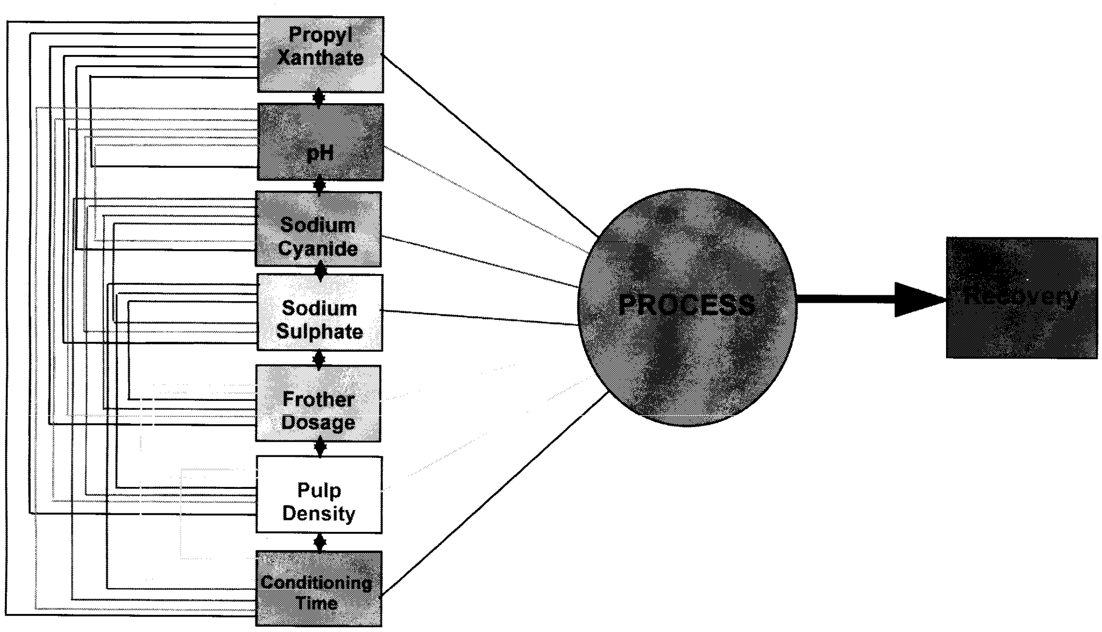

Fig. 2: Recovery model two for enrichment of copper ore of North Wazirstan.

Model No. 3

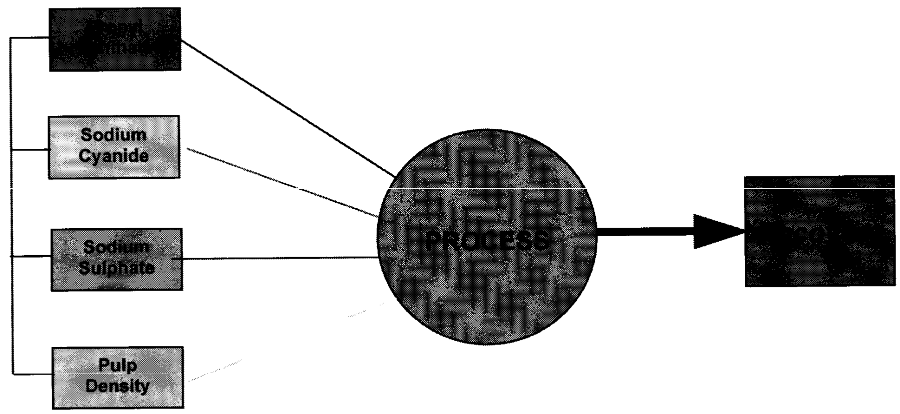

Fig. 3: Recovery model three for enrichment of copper ore of North Wazirstan.

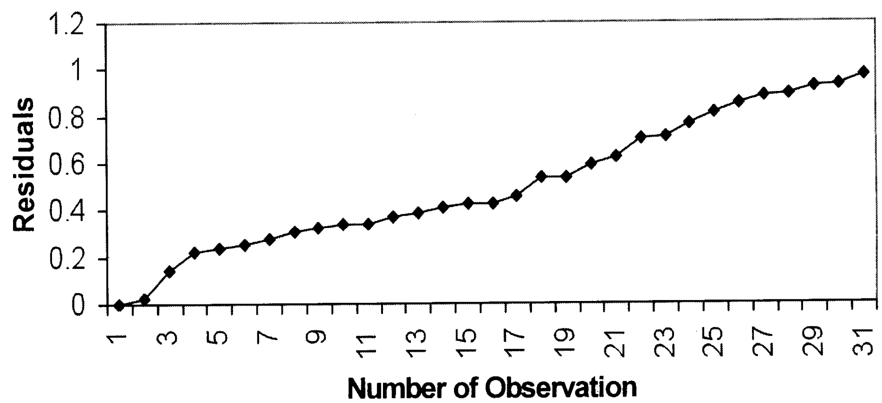

Fig.4: Visual normal test for standard residuals for seven process parameters.

Table-3: Test for Normality of Residuals.

Bin	Frequency	Cumulative %
-2	1	3.33 %
-1	2	10.00 %
0	14	56.67 %
1	7	80.00 %
2	6	100.00 %
3	0	100.00 %
More	0	100.00 %

Jarque–Bera: A Combined Test

Instead of using the two tests separately, one can use a linear combination of the two. The Jarque Bera test was devised as an optimal test against a certain class of alternatives to the null distribution. The test statistic is:

$$JB = T\{EK^2/24 + (SK)^2/6\}$$

In this study the value of Jarque – Bera (JB) is 2.747; in the Table-4 given below here are calculated values of different tests and also their critical values calculated by simulation.

Table-4: Test for Normality of Seven Variables.

Test	Calculated	Lower critical value	Upper critical value	Results
Skew ness	-0.47	-.7	.7	Pass
E.Kurtosis	1.1	-.99	1.55	Pass
Jarque-Bera	2.747	N.A	4.62	Pass

From the standard residual plot Fig. 6, it is clear that approximately 68 % data is inside the

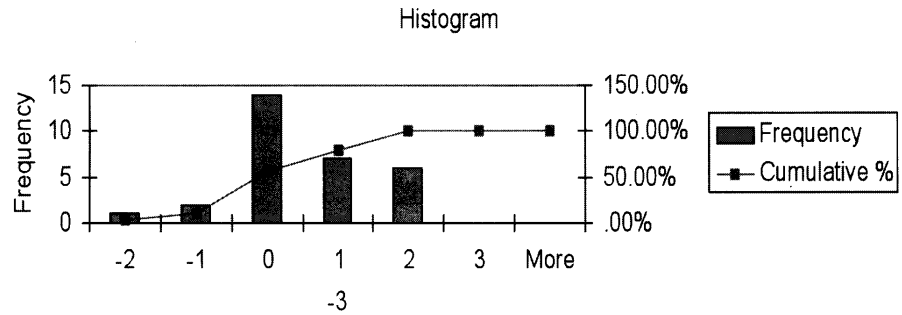

Fig. 5: Histogram of seven variables.

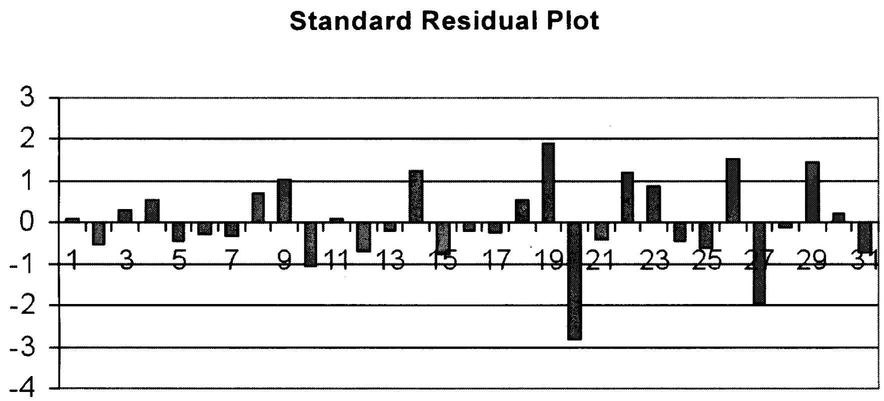

Fig. 6: Showing standard residual plot for seven variables.

closed interval between –1&1 and approximately 95 % data is in the closed interval –2 & 2, which is a sign of normality. If we look it more deeply we can see that the variation in the first half is different than the second half, which is indicating heteroscedasticity problem.

Testing for Heteroscedasticity

Let SER (1) and SER (2) be the Standard Error of Regression for the first half and the second half of the data set respectively. If the ratio SER(1)/SER(2) is close to 1 then the SE's on both halves of the data set are similar. If the ratio is for from 1 than the two SE's are different. The Goldfled-Quandt statistic is based on the ratio of variances (not SE's);

$$GQ = [SER(2) / SER(1)]^2$$

Now Var1 (Variance of first half) = 0.654, Var2 (Variance of second half) = 1.298, GQ test = 0.254, p-value = 0.995. The value of GQ test is 0.28, which is very much different from 1. This can also be seen from the p-value of GQ test. Standard residuals are normal but are not identically distributed, so it fails to be i.i.d random variables. Thus R^2 is meaningless.

The following is our new model, without intercept, which consists of four significant variables:

$$Y_R = \beta_1 X_1 + \beta_2 X_3 + \beta_3 X_4 + \beta_4 X_6 + \varepsilon \text{ ....(2)}$$

In Table-5 the value of R-square is approximately the same but the value of standard error has increased a little bit. Similarly the F statistic and the p-value suggests that all variables are collectively important. The significance F level increased which is a healthy sign. The p-value of t-statistic of each variable shows that all variables are individually important.

Fig. 7 shows that through it has little deviation from 45 degree line yet it does not give vital evidence against the normality.

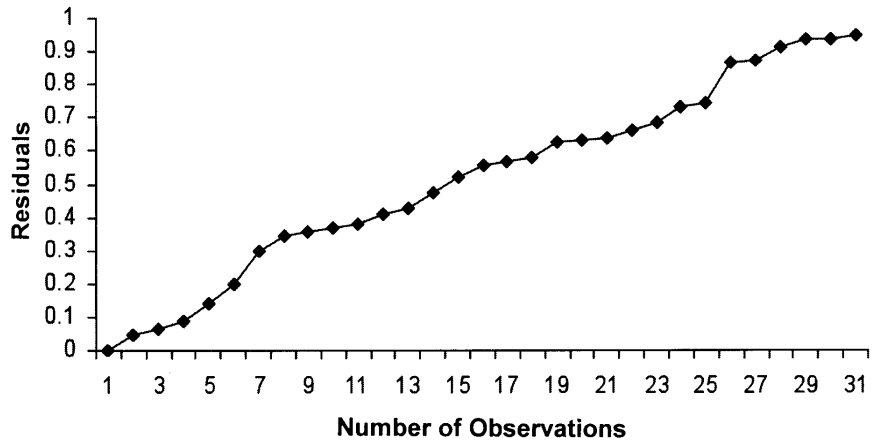

Fig. 7: Normality test for standard residuals.

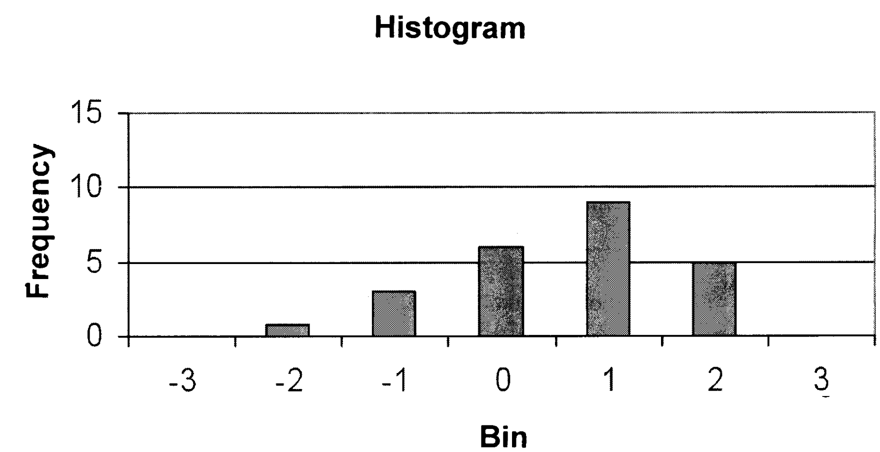

Fig. 8: Showing histogram of four significant variables.

Table-5: OLS Estimates for Four Significant Process Parameters.

	Coefficients	Standard Error	t. Statistic	P-value
Intercept	0	#N/A	#N/A	#N/A
X_1	0.060	0.019	3.226	0.003
X_3	0.770	0.089	8.683	3E-09
X_4	0.170	0.035	4.814	5E-05
X_6	0.924	0.114	8.115	1E-08
R-square	0.93	F-statistic	98.31	
Adjusted R	0.89	Standard Error	3.76	
Observations	31			

Test for Normality

Table-6 shows that Skewness = -0.62, Result is pass and E. Kurtoses is Pass. Frequency table-3 and the histogram Fig. 8 shows that data is normal. Another way of checking the normality is shown in Fig. 9. Approximately 68 % data is inside –1 to 1 and 95 % data is in the interval –2 to 2, which is the evidence that data is normal.

For identical distribution we again use the GQ test and here *Var-1* = 0.8, Var-2 = 1.21, GQ Test = 0.43, and p-value = 0.94. It qualifies the GQ test. So residuals are identically distributed. Correlation test suggest that residuals are independent from each other. Therefore we can conclude that residuals are i.i.d normal.

To estimate variances consistently, it is essential to have a model for them, of the type postulated in the Breusch-Pagan test. Thus for the second purpose listed above, improvement of

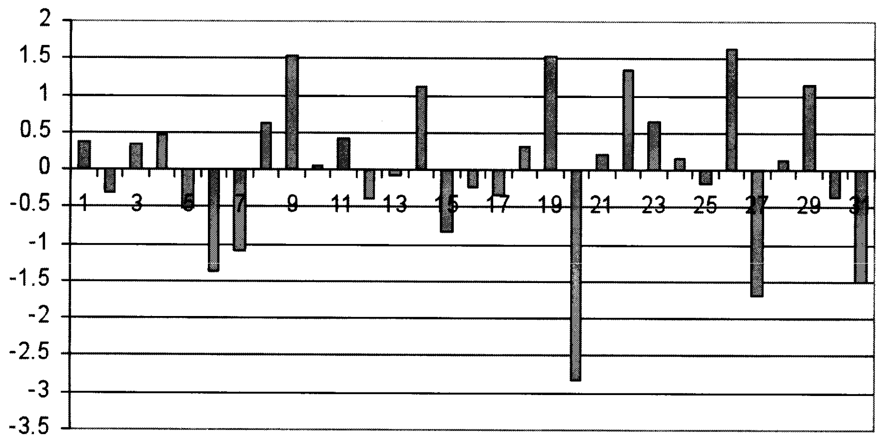

Fig. 9: Showing standard residual plot for four variables.

Table-6: Test for Normality for Four Variables.

Test	Calculated value	Lower critical value	Upper critical value	Result
Skew ness	-0.6	-.7	.7	Pass
Kurtosis	E-.89	-.99	1.55	Pass
Jarque bera	-1.8	N.A	4.62	Pass

efficiency of estimation, the Breusch-Pagan test is the optimal test. Once the test shows us which regressors are important determinants of variances, using the auxiliary equation to estimate variances and construct an FGLS estimator. The chow test value is 2.015 and this model is structurally stable.

We drop few variables and whether our new model is better than before. We perform F-test to see if the removal of the three least important regressors, X_2, X_5, X_7 has made any significant difference. The F-statistic can be computed as follows:

F = [(CRSS-URSS)/2]/[URSS/(T-K)]
= [(382.5-311.05)/2]/[311.05/(31-4]
= 3.1004

CRSS (Constrained Residual Sum of Square) = 382.5

URSS (Un-constrained residual sum of square) = 311.05

K = Number of regressors = 4

T = Number of observations = 31

This has degrees of freedom 2 and 27 using FDIST (3.1,2,27) we get the p-value 0.0641 or 6.41 %, which is not significant. This means that we can drop the three regressors from first model. It has a simpler theoretical structure. By using this model one can estimate Y_R within $\pm$3.704488 with 68 % probability and $\pm$7.4 with 95 % probability result.

Experimental

The data was recorded in the series of seven experiments with a total of 31 different treatments to evaluate the flotation response using different dosages and type of collector. The experiments were carried out in the Department of Mining Engineering, N.W.F.P, University of Engineering & Technology Peshawar. Experiments were carried out to observe the combined effect of propylxanthate, pH, sodium cyanide, sulfidizer, frother dosage, pulp density and conditioning time in the laboratory on the recovery of final concentrate of copper.

Conclusion

From this study we conclude that the appropriate model consists of four explanatory variables *i.e.* collector type and Dosage (X_1). Depressant sodium cyanide NaCN(X_3), Sulfidizer Na_2S(X_4), and Pulp Density (X_6).

Therefore our suitable recovery model is:

$$Y_R = 0.060X_1 + 0.770X_3 + 0.170X_4 + 0.924X_6 + \varepsilon \;..(3)$$

It is clear from the model that curve passes through the origin. It is obvious from this model that if we increase one unit of X_1, Y_R will increase 0.06 unit keeping all other variables constant. We can define all other coefficients in the same fashion. These coefficients (slopes) give partial values.

References

1. S. M. Badshah, *Record of FATA Development Corporation*, **11**, 2 (1983).
2. S. M. Badshah, *Record of FATA Development Corporation*, **3**, 14 (1985).
3. M. M. Khan, K. G. Jadoon and Amanullah Khan, *Proceeding of Seminar on Prospects and Problems of Mineral Based Industry in Pakistan*, NWFP University of Engineering and Technology, Peshawar, (1984).
4. M. M. Khan, *Science Vision*, **6**, 10 (2000).
5. Z. Wang, Lecture on *Distribution and Metallogenic Models of Large Copper Deposits of The World In North Waziristan Copper Ore*, at NWFP University of Engineering and Technology Peshawar (1996).
6. I. Huber-Panu, E. Ene-Danalance, D. J. Cojocariu, In D. W. Fuerstenau, *Flotation, A.M. Guardian Memorial*, **AIME**, p. 675 (1976).
7. J. A. Herbst and A.V. Potapov, *Journal of Mineral Metallurgical Processing*, **21**, 57 (2004).
8. R. R. Klimpel, *Int. Journal Mineral Processing*, **58**, 71 (2000).
9. H. Kuopanportti, T. Suorsa, O. Dahl, and J. Niinimaki, *Int. Journal Mineral Processing*, **59**, 327 (2000).
10. S. G. Malghan, *Journal of Mineral Engineering*, p. 905 (1989).
11. E. C. Cilek, *Journal of Mineral Engineering*, **17**, 81 (2004).
12. M. Barbaro and L. Piga. *Journal of Mineral Engineering*, **12**, 355 (1999).
13. J. F. Oliveira, S. M. Saraiva, J. S. Pimenta, and A. P. A. Oliveria, *Journal of Mineral Engineering*, **14**, 99 (2001).
14. M. Polat and S. Chander, *Int. Journal of Mineral Processing*, **58**, 145 (2000).
15. B. A. Wills, *Mineral Processing Technology*, Butterworth Heinemann Oxford Press, p. 284 (1997).
16. M. Xu, *Journal of Mineral Engineering*, **11**, 271 (1998).
17. R. D. Crozier, *Journal of Mineral Engineering*, **4**, 839 (1992).