Modellıng, Compressive Strength of Standard Cem-I 42.5 Cement Produced in Turkey wıth Stepwıse Regressıon Method

[1]ABDULKADIR TEPECIK*, [2]ZEHRA ALTIN AND [2]SEYFETTIN ERTURAN
YILDIZ Technical University, Davutpasa ESENLER 34210 ISTANBUL, Turkey.

(Received 15th July 2008, accepted in revised form 6th November 2008)

Summary: A multiple linear regression model was developed for the prediction of the 2, 7, and 28-day compressive strength of CEM-I 42.5 produced in Turkey Cement Fabrics. Attention has been paid to the right choice of independent variables involved in this model, especially the characteristics of the cement itself.

Different combinations of variables were introduced into the model, in order to choose the variables that can properly predict the compressive strength of the cement.

Multiple Linear Regression (MLR) analyses with backward stepwise were performed to describe the relationships between the compressive strength values and the chemical or physical properties of the cement. The advantage of using this technique lies in the fact that it deals simultaneously with several variables. The analysis is designed to see which factors are significant in explaining the compressive strength of the cement.

The evaluation of the proposed model was performed by various statistical tests, all of which were successful. These statistical tests included: multiple correlation, test of the significance of coefficients (t-test), estimation of confidence intervals for coefficients, conditional sums of squares, R-squared and analysis of variance.

Models obtained this way can predict the compressive strength of the cement with very small standard errors and coefficients of correlation of 0.9961 and 0.9955, and 0.9983, for cement strengths at 2, 7 and 28 days, respectively. There was very good agreement between the strength predicted by the multiple regression model and the experimental results. These models explained 99 % of the variability in strengths.

Introduction

Both the producers and the purchasers want cements which are suitable and comply with the specifications all around of the world [1]. The specifications, generally, include a statement of physical and chemical requirements [2]. The mechanical strength of the cement in the set and hardened conditions are the most obviously required values for structural usage [3, 4]. Strength of cement, as specified by all the standards, is very important (from 2 to 28 days). The traditional 28 days standard test has been found to give a general index of the overall quality and acceptance of concrete and has served well for so many years. This age has already been introduced in most of the international standards like the British and ASTM standards.

Many efforts have been made to identify the relationship between various physicochemical and mineralogical parameters of cement clinker and the strength of the cement [5]. Although such relationships have not been fully justified [4], many models have been proposed for the prediction of the compressive strength of CEM-I 42.5 [6-13]. A carefully designed model can serve as a valuable tool during the production of cement and the control of its quality.

In the present work, Multiple Linear Regression (MLR) models were developed for the prediction of the 2, 7 and 28-day compressive strength of CEM-I 42.5 using the backward stepwise method. These models are useful to predict the compressive strength of cement when it has various particle sizes and known chemical and mineralogical properties.

The evaluation of the models was performed by various statistical tests listed above [14-16]. The important factors expressing compressive strength of cement were identified [17]. This research is significant as the availability of such a model would possibly provide a hard balance and equality, between controlling the quality and the economics of cement production.

*To whom all correspondence should be addressed.

Nomenclature

CEM-I 42.5	: Portland Cement Type I	R90	:	Particle size under 90 μ above 40 μ
LSF	: Lime Saturation Factor	MLR	:	Multiple Lineer Regression
C_3S/C_2S	: Alite blite ratio	LH	:	Lost of Heating
C_3A/C_4AF	: Aluminate Ferrite ratio	S2	:	Compressive Strength for 2-day
Sb	: Blaine	S7	:	Compressive
R40	: Particule size under 40 μ	S28	:	Compressive Strength for 28-day

Results and Discussion

Regression Models

The analysis of variance is used in regression analysis to test for the significance of the overall model by the F-test. We used Student's T tests to determine if the partial regression coefficient for each independent variable represents a significant contribution to the overall model.

The values of P less than 0.10 indicate statistically significant nonzero coefficients at a 90 % confidence level. There is a significant correlation between the pair of variables with 90 % confidence.

In summary, this paper provides a reliable method for strength estimation from Turkish CEM-I 42.5 productions. It is of great advantage because of its simplicity, accuracy and convenience. It is very important for Cements Researches. The Results are:

Estimated Coefficients

For 2 days: R^2 = 0.9961 R^2adj = 0.9935 F = 385, p-value = 2.33E-07

	Coefficients	Standard Error	t-Stat	p-value
C_3A/C_4AF	-30.66	9.45	-3.24	0.018
Sb	0.18	0.02	7.39	0.000
R40	-4.37	1.02	-4.30	0.005
R90	49.97	12.70	3.93	0.008

For 7 days: R^2 = 0.9955 R^2adj = 0.995 F = 2019, p-value = 6.69E-12

Estimated Coefficients

	Coefficients	Standard Error	t-Stat	p-value
Sb	0.12	0.00	44.94	0,000

For 28 days: R^2 = 0.9983 R^2adj = 0.9979 F = 2329, p-value = 8.64E-12

Estimated Coefficients

	Coefficients	Standard Error	t-Stat	p-value
LSF	0.28	0.14	2.04	0.076
Sb	0.07	0.04	1.92	0.092

The data analysis resulted in the following stepwise regression equations:

For age 2 days

$$S2 = -30.66(C_3A/C_4AF) + 0.18(Sb) - 4.37(R40) + 49.97(R90) \quad (1)$$

For age 7 days $S7 = 0.12(Sb)$

For age 28 days $S28 = 0.28(LSF) + 0.07(Sb)$

We extract the following consequences from above tables and equations:

For 2 days; In Sb 100 units raising will increase 18 units compressive strength for 2 days.

While the particle size in composition ($40\mu \le particle\,size \le 90\mu$) 100 units raising increases about 50 units compressive strength, particle size less than 40 μ in composition 100 units raising will decrease 437 units compressive strength. Lastly, If C_3A/C_4AF ratio increase 100 units, the compressive Strength will decrease 3066 units.

For 7 days; In Sb 100 units raising will increase 12 units compressive strength.

For 28 days; In Sb or LSF 100 units rising will increase 7 and 28 units compressive strength, respectively.

When the setting time increases according to model equations, the factors which create compressive strength concretize. The effectiveness of other factors are vanishing. The same manner, when the setting time arrives 28 days, again the effectiveness of blaine is continuing, it appears that as well LSF affects the compressive strength.

This study revealed that the adjusted correlation coefficients of (1), (2), and (3) multiple regression models are 0.9935, 0.9955, and 0.9979 in Turkey Cement Plants CEM-I 42.5 products. This clearly shows the importance of the introduction of these factors in the regressions.

The independent variables can explain about 99 % variation of compressive strength *via* these model equations.

Statistical Tests and Inferences

The F-Test statistics that evaluate overall significance are 385, 2019, 2329 for (1), (2), and (3) equations, respectively. These values are much higher than the F-statistics table values that we selected ($\alpha = 0.1$) for hypotheses test.

The obtained mathematical models sufficiency is made by Student's T-test. For each three models ninety percent of t distribution is closer to mean than we search t value.

In above results, all coefficients have $p < 0.10$. As can be seen, $p < 0.10$ for all pairs, indicating that all variables are independent from one another.

Average differences between measured and predicted compressive strengths are 0.11 %, 0.12 %, and 0.02 %, in other words 0.004MPa, 0.052MPa, and 0.09MPa for setting days, respectively.

Table-1 shows the measured and predicted compressive strength after 2, 7 and 28 days of cement. It seems that differences between measured and predicted values decreased as the numbers of day are increased. Results give the regression coefficients of the prediction model above for the prediction of 2, 7 and 28 days coefficient of correlation and standard error of estimate corresponding to each set of variables used in each model. In this table several notes have to be pointed out:

1. The effect of the four main compounds on the compressive strength of cement at the age of 7 days was more than their effect on the 28 days compressive strength. This could be clearer from the value of the standard error for the equation used in the prediction. This looks very reasonable, as the effect of these compounds at early ages is more affected by their rate of hydration, more than their effect at later ages (especially the silicates).
2. Effect of fineness (as expected) on the strength at 7 days is more pronounced than that at the age of 28 days, also, it can be seen that the introduction of fineness in model (2) reduces (slightly) the effect of the silicates. This could be because fineness is a strong effective factor that contributes to strength, especially at early age as 7 days.

In the proposed models, a fact was proved, this was the role of the variables affecting strength of cement is not additive (especially the four main compounds), but there is an interaction between them, *i.e.*, the introduction of one variable may enhance the effect of some variables and weaken the effect of other variables.

The multiple regression models that was developed in the present work was shown to satisfy all the statistical tests used for its evaluation. Fig. 1 shows a plot of the observed (experimental) values versus the values predicted from the fitted model. The proposed model has a very good predicting ability.

Table-1: Measured and Predicted Compressive Strength Values.

Specimen Number	Measured Compressive Strength (MPa)			Predicted Compressive Strength (MPa)			Difference (%)		
	2 d.	7 d.	28 d.	2 d.	7 d.	28 d.	2 d.	7 d.	28 d.
1	28.15	45.9	56	30.41	43.22	53.98	8.04	5.82	3.6
2	31.28	41.19	51.09	28	40.74	52.49	10.51	1.08	2.74
3	36.87	40.4	52.37	36.71	44.83	55.22	0.44	10.97	5.44
4	33.73	45.5	55.11	34.47	43.56	53.59	2.18	4.26	2.77
5	23.05	39.62	54.23	26.48	42.64	52.59	14.91	7.61	3.03
6	33.15	45.01	54.23	31.02	47.41	55.51	6.4	5.33	2.36
7	27.75	40.4	51.19	27.64	40.12	52.21	0.41	0.7	1.99
8	24.22	44.52	54.62	23.84	41.22	52.89	1.56	7.42	3.17
9	23.93	39.52	50.01	24.05	42.12	53.64	0.5	6.58	7.25
10	26.97	47.07	57.07	26.52	42.75	53.71	1.68	9.19	5.9

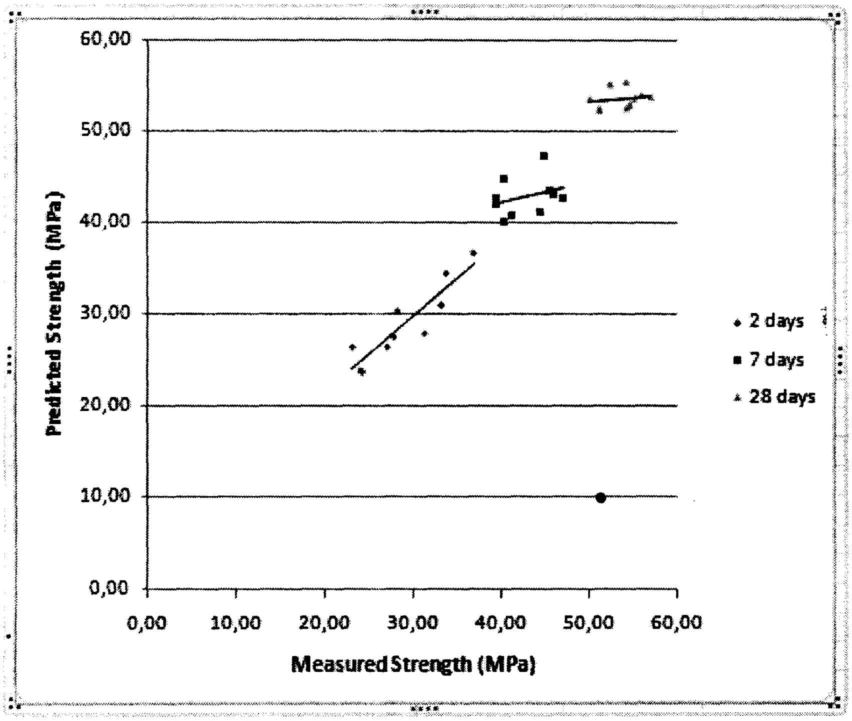

Fig. 1: Predicted versus observed compressive strengths of CEM-I 42.5 for 2, 7, and 28 days.

The choice of the independent variables of the model was based on statistical as well as physical considerations. A large number of variables with physical importance were considered and those satisfying statistical criteria given above were actually selected. The variable Sb is critical and also of high significance. This was expected, since the strength development under conditions of accelerated hardening provides a good measure of the actual hydraulic behavior of the cement. The C_3A content is the less significant variable among those used. It reflects the composition of the liquid phase of the clinker, and incorporates factors related to the burning process and the cooling rate.

The expected relationship between density of the mortar cubes and the compressive strength is a positive one, in other words increasing density increases compressive strength. The most possible explanation is the one concerned with hydration, as the chemical reactions proceed, the hydration will progress and there would be certainly a continuous formation of hydration products and thus, development in strength with reduction in porosity and consequently the density of the mass will be increased.

Experimental

Materials

Ten different CEM-I 42.5 samples were tested for the development of the model. The sampling was random and it was performed during a period of three months at the place of production. The chemical analysis for the specimens of CEM-I 42.5 is shown in Table-2a.

The potential mineral compositions of CEM-I 42.5 specimens are shown in Table-2b.

Other physical factors such as fineness (R90, R40), blaine are also shown in above Table-2c. The compressive strength of the CEM-I 42.5 samples were measured at the ages of 2, 7 and 28 days, according to EN specifications, and the results are shown in above Table-2c.

Table-2a: Chemical analysis of CEM-I 42.5 specimens.

	CHEMICAL ANALYSIS									
Specimen No.	SiO_2	AL_2O_3	FE_2O_3	CAO	MGO	K_2O	NA_2O	SO_3	LH	SCAO
1.	19.35	4.84	3.63	63.17	1.42	0.83	0.12	2.91	3.90	1.5
2.	19.55	4.64	3.70	64.19	2.4	1.02	0.68	3.81	2.51	1.5
3.	19.58	5.25	3.67	64.71	2.13	0.48	0.08	2.65	2.37	1.5
4.	20.06	4.65	3.73	63.49	2.64	0.12	0.23	2.75	3.18	1.5
5.	20.48	4.83	3.56	63.85	2.23	0.44	0.16	2.94	3.13	1.5
6.	20.04	5.06	3.53	62.99	2.31	0.35	0.23	2.94	3.12	1.5
7.	19.27	5.15	3.30	63.42	2.47	0.8	0.74	3.02	3.29	1.5
8.	19.29	4.54	3.39	62.91	2.62	0.63	0.47	3.10	2.77	1.5
9.	19.40	4.97	3.42	64.26	2.85	0.62	0.46	3.21	3.32	1.5
10.	19.52	4.96	3.36	63.83	2.83	1.15	0.41	3.18	2.78	1.5

Table-2b: Mineralogical analysis of CEM-I 42.5 specimens.

	MINERALOGICAL ANALYSIS							
Specimen No.	SM	KUHL	C_3S	C_2S	AR	C_3A	C_4AF	LSF (%)
1.	2.28	101.81	66.22	8.16	1.33	6.84	11.04	98.05
2.	2.34	102.81	70.1	5.86	1.25	6.19	11.25	98.1
3.	2.2	102.44	67.93	7.56	1.43	7.86	11.16	98.97
4.	2.39	99.37	63.26	12.5	1.25	6.17	11.34	95.94
5.	2.44	98.01	60.57	15.61	1.36	6.93	10.82	94.41
6.	2.33	98.19	58.91	15.58	1.43	7.58	10.73	94.5
7.	2.28	102.4	66.24	7.68	1.56	8.2	10.03	98.43
8.	2.43	102.49	67.98	6.49	1.34	6.44	10.31	98.5
9.	2.31	103.34	69.7	5.52	1.45	7.53	10.4	99.21
10.	2.35	102.19	67.2	7.71	1.48	7.6	10.21	98.11

The preparation of the specimens was performed according to EN specifications. Physical, chemical, and mineralogical analysis was fulfilled by Oxford Labx 3000 X-Ray spectrophotographmeter device. A sample of the SEM picture of CEM-I 42.5 is given below in Fig. 2.

Fig. 2: CEM-I 42.5 SEM picture.

Preparation of Samples and Methods

For physical tests, selected 40 μ and 90 μ under sieve cement samples are used in EN 197 standards. Initial and final setting times were performed in laboratory conditions. Vicat device was used to identify normal density of the cement. Density definitions were made by Le Chatelier method. Toni Technick Blaine device was used for specific surface determination. The results obtained are shown in Table-2a, b, and c.

Modeling of the Compressive Strength of Cement with Statistical Methods

Determination or prediction of the strength of cement could be attained by suitable mathematical models (MLR) with independent variables affecting the strength (dependent) development of cement.

Predictor or independent variables used in the mathematical models in this work are:

Table-2c: Physical analysis of CEM-I 42.5 specimens.

	PHYSICAL ANALYSIS									
	Compressive Strength (MPa)			Fineness (%)		Specific Surface			Setting Time	
Specimen No.	2-day	7-day	28-day	40 μ	90 μ	Density (kg/m³)	Blaine (m²/kg)	Water (%)	Initial (Min.)	Final (Min.)
1	28.15	45.9	55.96	6.5	0.3	3150	353.9	28.8	200	270
2	31.28	41.19	51.09	9	0.5	3170	333.6	29	250	310
3	36.87	40.4	52.37	5	0.3	3180	367.1	28.8	180	220
4	33.73	45.5	55.11	8.5	0.5	3170	356.7	28.5	220	310
5	23.05	39.62	54.23	8.2	0.4	3160	349.1	28.3	270	310
6	33.15	45.01	54.23	6	0.2	3180	388.2	28.8	240	290
7	27.75	40.4	51.19	7	0.5	3140	328.5	29	230	300
8	24.22	44.52	54.62	7.3	0.3	3160	337.5	28.8	250	300
9	23.93	39.52	50.01	8	0.4	3160	344.9	28	260	320
10	26.97	47.07	57.07	7.5	0.4	3160	350	28.8	260	310

Lime saturation Factor (LSF).

Two main ratios for the four main compounds of CEM-I 42.5 (C_3S/ C_2S and C_3A/C_4AF).
Specific Surface (Blaine = Sb).
Fineness or particle size of cement (R40, R90)

These factors can affect the strength of the cement. Thus suitable proposed models should be able to predict the strength of CEM-I 42.5 at the age of 2 days, and 7 days as well as the age of 28 days.

Before the determination of the coefficients of the model, it is necessary to ensure that the chemical and physical descriptors (independent variables) given above do not correlate well with each other. In practice the question is whether there is a significant correlation among the independent variables. Table-3 was constructed with the correlation coefficients between pairs of variables.

Table-3: Correlation coefficients for physical descriptors of cements.

	LSF	C_3S/C_2S	C_3A/C_4AF	Sb	R40	R90
LSF	1.000	0.901	0.254	-0.519	-0.128	0.168
C_3S/C_2S		1.000	0.073	-0.603	0.170	0.239
C_3A/C_4AF			1.000	0.005	-0.480	-0.707
Sb				1.000	-0.576	-0.707
R40					1.000	0.732
R90						1.000

It can be seen clearly from this table why the C_3S/C_2S ratio was not included into the models. C_3S/C_2S ratio and LSF must be independent of each other to be included in the model. However, there is a high correlation coefficient above 90 % between these two variables. That is, the value of correlation coefficient between C_3S/C_2S ratio and LSF proves that they are relevant to each other. Thus C_3S/C_2S ratio can not be selected as an independent variable into model if LSF is to be included in the same model.

For each independent variable, several simple model types (such as $x, \frac{1}{x}, Log(x), \sqrt{x}$) were tested and the form giving the best correlation was selected for MLR analyses. The multivariable linear equations were found to be very suitable for predicting strength of CEM-I 42.5. These model equations can be produced a reliable relationship between the strength of the cement and its own characteristics (depending on the above mentioned variables). In stepwise regression analysis, we begin the analysis, with all the variables under consideration included in the model, and then (possibly) we remove one of the variables in each step (backward and forward method). These are two of several available approaches in choosing the "best" set of independent variables for the model.

In this study, MLR with backward stepwise technique was used to study the dependence of strength variable on the independent variables (LSF, C_3S/C_2S, C_3A/C_4AF, Sb, R40, and R90). The determination of the coefficients of the model and all statistical tests were performed using the statistical computer package. In principle, extrapolation is not acceptable in this kind of experimental model. Likely variables, for calculations according to above studies, are shown Table-4.

Table-4: Selected variables for calculations.

Specimen Number	LSF (%)	C_3S/C_2S	C_3A/C_4AF	Sb (m^2/kg)	R40 (μ)	R90 (μ)
1	98.05	8.12	0.62	353.9	6.5	0.3
2	98.1	11.96	0.55	333.6	9	0.5
3	98.97	8.99	0.7	367.1	5	0.3
4	95.94	5.06	0.54	356.7	8.5	0.5
5	94.41	3.88	0.64	349.1	8.2	0.4
6	94.5	3.78	0.71	388.2	6	0.2
7	98.43	8.63	0.82	328.5	7	0.5
8	98.5	10.47	0.62	337.5	7.3	0.3
9	99.21	12.63	0.72	344.9	8	0.4
10	98.11	8.72	0.74	350	7.5	0.4
Min	94.41	3.78	0.54	328.50	5.00	0.20
Max	99.21	12.63	0.82	388.20	9.00	0.50
Average	97.42	8.22	0.67	350.95	7.30	0.38
Standard Deviation	1.79	3.13	0.09	17.37	1.22	0.10
Standard Error	0.32	0.98	0.00	30.19	0.15	0.00

In this table while the ratio C_3S/C_2S is included for completeness it is not used in the analysis.

Now it has been determined which variables are affecting equations with best correlation values, via a stepwise regression technique. Next the best model equations have been rewritten with coefficients. We can proceed now to calculate the coefficients and test of significance and confidence intervals (Table-5).

Table-5: Selected variables for the prediction of compressive strength of CEM-I 42.5 with stepwise regression analysis.

	Strength		
Variables/Days	2 days	7 days	28 days
LSF			√
C_3S/C_2S			
C_3A/C_4AF	√		
Sb	√	√	√
R40	√		
R90	√		

Conclusions

The following conclusions were drawn from this study:

Three algorithms were proposed to predict the strength of cement for different ages. These algorithms predict the compressive strength of cement at the ages of 2, 7 and 28 days. In the development of these algorithms regressions multivariable relationships were used. This type of regressions proved to yield better predicted results than the multivariable linear regressions used by other researches. Each three models can contain up to six variables. These variables were divided into combinations, which included the following variables:

Chemical composition ratios of main cement compounds and fineness (particle size),

Specific Surface (blaine),
Lime Saturation Factor.

The total number of independent variables used were six. The best correlation coefficients obtained were 0.99, 0.9983 and 0.9955 for 2, 7, and 28 days, respectively. The blaine variable yielded excellent information on the cement compressive strength.

From the comparison between the regressions obtained, it can be said that including lime saturation factor compounds did not result in significant improvement in the correlation coefficients. Therefore these variables may not be included in the regression.

The developed multiple regression models were shown to be able to predict with adequate accuracy the 28-day compressive strength of cement. The models were evaluated by various statistical tests, which were all successful. The usefulness of the proposed model to the cement industry is based on its ability to make predictions for the 28-day strength of the cement. This gives the opportunity to civil engineers to make the necessary adjustments to improve the quality of the product in due time.

References

1. ASTM C 125, *Standard Terminology Relating to Mortar and Mortar Aggregate*, Annual Book of ASTM Standards, (2006).
2. F. M. Lea, *Chemistry of Cement and Concrete, 3rd Ed.*, Edward Arnold Ltd., London (1970).
3. R. L. Blain, H. T. Ami and M. R. Defore, *Compressive Strength Test of Mortars* Section 7, Interrelations Between Cement and Concrete Properties, Part 3, Building Science Series 8, U.S. Dept. of Commerce and National Bureau of Standards, Washington D.C., p. 1 (1968).
4. TS EN 196, *Methods of Testing Cements-Determination of Strength*, Turkish Standards. Institution, Ankara (2002).
5. H. Braggemann and L. Brentup, *Translation Zement-Kalk-Gips* **1**, 30 (1990).
6. L. P. Aldridge, *Proceedings of the 7th International Congress on the Chemistry of Cement*, **VI**, Paris, 83 (1980).
7. K. Alexander, *Cement and Concrete Research*, **2**, 663 (1972).
8. R. Hargrave, D. Venkateswaran, V. Deshmukh and A. Chatterjee, *Proceedings of the 8th International Congress on the Chemistry of Cement*, **II**, Rio de Janeiro, 167 (1986).
9. L. Petrasinovic and M. Duric, *Cement and Concrete Research*, **20**, 484 (1990).
10. S. Popovics, *Proceedings of the 7th International Congress on the Chemistry of Cement*, **VI**, Paris, 47 (1980).
11. S. Popovics, *Cement and Concrete Research*, **17**, 821 (1987).
12. M. Relis, W. Ledbetter and P. Harris, *Cement and Concrete Research*, **18**, 674 (1988).
13. S. Tsivilis and G. Parrisakis, *Cement and Concrete Research*, **25**, 9 (1995).
14. R. R. Hocking, *Methods and Applications of Linear Models. Regression and the Analysis of Variance*, Wiley NY (1996).
15. D. F. Morrison, *Multivariate Statistical Methods 3rd Ed.* McGraw-Hill NY (1990).
16. J. Neter, W. Wasserman and M. H. Kutner, *Applied Linear Statistical Models: Regression, Analysis of Variance, and Experimental Designs*, Irwin Homewood, IL, (1985).
17. A. M. Neville, *Properties of Concrete*, *4th Ed.* Pitman Publishing, London (1995).